

Meta-stable dynamical supersymmetry breaking near points of enhanced symmetry

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ABSTRACT: We construct a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. Our model consists of two sectors coupled by a singlet and combines dynamical supersymmetry breaking with an O’Raifeartaigh mechanism in terms of confined variables. The metastable vacua appear along a pseudo-runaway direction near a point of enhanced symmetry as a result of a balance between non-perturbative and perturbative quantum effects. We show that metastable supersymmetry breaking is a rather generic feature near certain enhanced symmetry points of gauge theory moduli spaces.

KEYWORDS: Supersymmetry Breaking, Supersymmetric gauge theory, Supersymmetric Effective Theories, Supersymmetry and Duality.

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1. Introduction

The idea that our universe may be in a long-lived metastable state in which supersymmetry is broken has recently led to an increased interest in developing models of supersymmetry breaking. This has opened many new possibilities in constructing field theory and string theory models.

On the field theoretic side, the work of Intriligator, Seiberg and Shih (ISS) [1] presented calculable metastable vacua using Seiberg duality. This motivated related field theory constructions, involving gauge mediation [2], generalized O’Raifeartaigh models [3], retrofitting [4], adjoint matter [26], applications to particle physics [5], etc. Similar developments have been seen in string theory based on a number of different tools, such as intersecting or wrapping branes [6], flux compactifications [7], Calabi-Yau’s with particular geometric properties [8], IIA/M-theory configurations [9] and others. Statistical analyses of the supersymmetry breaking scale on the landscape of effective field theories were done, for instance, in [10].

The ISS model consists of SQCD in the free magnetic range, and metastable vacua appear after taking into account one-loop corrections that lift the pseudo-moduli. Their work suggests that nonsupersymmetric vacua are rather generic, if one requires them to be only local, rather than global, minima of the potential. The construction still contained relevant couplings in the form of masses for the quarks though, and the search for models with all the relevant parameters generated dynamically has proven difficult; see [11–13], for recent work in this direction.

One lesson from ISS is that certain properties of moduli spaces can hint at the existence of metastable vacua. In their case, it was the existence of supersymmetric vacua coming in from infinity that signaled an approximate R-symmetry. Here we will point out that one should also look for another feature, namely, enhanced symmetry points, which are defined by the appearance of massless particles. We claim that if the moduli space has certain coincident enhanced symmetry points, metastable vacua with all the relevant couplings arising by dimensional transmutation may be obtained.

Let us motivate this claim. In order to generate relevant couplings dynamically, a gauge sector is required, which gives nonperturbative contributions to the superpotential. However, in general this leads to a runaway behavior. We will show that starting with two gauge sectors, the runaway may now be stabilized by one loop effects from the additional gauge sector, but only around enhanced symmetry points where quantum corrections are large enough. Such runaways which are stabilized by perturbative quantum corrections will be called ‘pseudo-runaways’. Surprisingly, the gauge theories where this occurs turn out to be generic.

The model considered here consists of two SQCD sectors, each with independent rank and number of flavors, coupled by a singlet. It involves only marginal operators with all scales generated dynamically. At the origin of moduli space, the singlet vanishes and the quarks of both sectors become massless simultaneously. There are thus two coincident enhanced symmetry points at the origin. While one of the SQCD sectors is in the electric range and produces a runaway, the other has a magnetic dual description as an O’Raifeartaigh-like model. Near the enhanced symmetry point, the Coleman-Weinberg corrections stabilize the nonperturbative instability producing a long-lived metastable vacuum. A feature of our model is that it may be possible to gauge parts of its large global symmetry to obtain renormalizable, natural models of direct gauge mediated supersymmetry breaking with a singlet. R-symmetry is broken both spontaneously and explicitly in our model.

The plan of the paper is as follows. In section 2, our model is introduced and its supersymmetric vacua are studied. In section 3, we analyze in detail the non-supersymmetric vacua and argue that they are parametrically long-lived. In section 4, we give a detailed analysis of the particle spectrum and the R-symmetry properties. In section 5, we argue that such metastable vacua may be generic near points of enhanced symmetry in the landscape of effective field theories. In section 6, we give our conclusions.

2. The model and its supersymmetric vacua

We consider models with two supersymmetric QCD (SQCD) sectors characterized by (N_c, N_f, Λ) and (N'_c, N'_f, Λ') , respectively, that are coupled to the same singlet field Φ . The field Φ provides the mass of the quarks in both sectors. In section 2.1, the general properties of such models will be discussed and their global symmetries analyzed. In section 2.2, we analyze the supersymmetric vacua. Section 2.3 will discuss for which range of the parameters (N_c, N_f, Λ) and (N'_c, N'_f, Λ') metastable vacua will be shown to exist. The upshot will be that one sector has to be taken in the electric range and the other sector in the free magnetic range.

2.1 Description of the model

The matter content of the models considered here consists of two copies of supersymmetric QCD, each with independent rank and number of flavors, and a single gauge singlet chiral superfield:

$$\begin{array}{cccc}
 & \text{SU}(N_c) & \text{SU}(N'_c) & \\
 Q_i & \square & 1 & i = 1, \dots, N_f \\
 \bar{Q}_i & \bar{\square} & 1 & \\
 P_{i'} & 1 & \square & i' = 1, \dots, N'_f \\
 \bar{P}_{i'} & 1 & \bar{\square} & \\
 \Phi & 1 & 1 &
 \end{array} \tag{2.1}$$

The most general tree-level superpotential with only relevant or marginal terms in four dimensions for the matter content (2.1) with $N_c, N'_c \geq 4$ is

$$W = (\lambda_{ij}\Phi + \xi_{ij})Q_i\bar{Q}_j + (\lambda'_{i'j'}\Phi + \xi'_{i'j'})P_{i'}\bar{P}_{j'} + w(\Phi), \tag{2.2}$$

where $w(\Phi)$ is a cubic polynomial in Φ . Remarkably, we shall find metastable vacua even in the simplest case of $w(\Phi) = 0$, which we assume from now on. The general situation is discussed in section 5 (in [12], the case $w(\Phi) = \kappa\Phi^3$ was used to stabilize Φ supersymmetrically).

At the classical level, the superpotential with $w(\Phi) = 0$ has an $U(1)_R \times U(1)_V \times U(1)'_V$ global symmetry under which the fields transform as

$$\begin{array}{cccc}
 & U(1)_R & U(1)_V & U(1)'_V \\
 Q_i & +1 & +1 & 0 \\
 \bar{Q}_i & +1 & -1 & 0 \\
 P_{i'} & +1 & 0 & +1 \\
 \bar{P}_{i'} & +1 & 0 & -1 \\
 \Phi & 0 & 0 & 0 \\
 \Lambda^{3N_c-N_f} & 2N_c & 0 & 0 \\
 \Lambda'^{3N'_c-N'_f} & 2N'_c & 0 & 0
 \end{array} \tag{2.3}$$

where the normalizations of the $U(1)_V \times U(1)'_V$ charges are arbitrary. In the quantum theory the $U(1)_R$ symmetry is anomalous with respect to the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics. The theta angles θ and θ' transform inhomogeneously under $U(1)_R$, and the holomorphic dynamical scale,

$$(\Lambda/\mu)^{3N_c-N_f} = e^{-8\pi^2/g^2(\mu)+i\theta}, \tag{2.4}$$

and likewise for $\Lambda'^{3N'_c-N'_f}$, transform with charges given in (2.3). The $U(1)_R$ symmetry is broken explicitly by the anomalies to the anomaly free discrete subgroups $Z_{2N_c} \subset U(1)_R$ and $Z_{2N'_c} \subset U(1)_R$, respectively. The largest simultaneous subgroup of both Z_{2N_c} and $Z_{2N'_c}$ which is left invariant by the superpotential (2.2) which couples the two gauge sectors through Φ interactions is $Z_{\text{GCD}(2N_c, 2N'_c)} \subset U(1)_R$, where $\text{GCD}(2N_c, 2N'_c)$ is the greatest common divisor of $2N_c$ and $2N'_c$.

In the $SU(N_f)_V \times SU(N'_f)_V$ global symmetry limit the superpotential (2.2) (with $w(\Phi) = 0$) reduces to

$$W = (\lambda\Phi + \xi)\text{tr}(Q\bar{Q}) + (\lambda'\Phi + \xi')\text{tr}(P\bar{P}). \tag{2.5}$$

This superpotential has the same $U(1)_R \times U(1)_V \times U(1)'_V$ global symmetry as (2.2), as well as a $Z_2 \times Z_2$ conjugation symmetry under which $Q_i \leftrightarrow \bar{Q}_i$ and $P_i \leftrightarrow \bar{P}_i$, respectively. The form of the superpotential (2.5) may be enforced for any N_c and N'_c by weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. One of the masses, ξ or ξ' , may always be absorbed into a shift of Φ . For $\xi = \xi'$ both masses may simultaneously be absorbed into a shift of Φ , and the tree level superpotential in this case reduces to

$$W = \lambda\Phi \text{tr}(Q\bar{Q}) + \lambda'\Phi \text{tr}(P\bar{P}). \tag{2.6}$$

This form agrees with the naturalness requirement that there be no relevant couplings. $\Phi = 0$ is an enhanced symmetry point for both sectors, where the respective quarks become massless. The case $\xi \neq \xi'$ is analyzed in section 5.

At the classical level this superpotential has an $U(1)_R \times U(1)_A \times U(1)_V \times U(1)'_V$ global symmetry

	$U(1)_R$	$U(1)_A$	$U(1)_V$	$U(1)'_V$	
Q_i	+1		$-\frac{1}{2}$	+1	0
\bar{Q}_i	+1		$-\frac{1}{2}$	-1	0
$P_{i'}$	+1		$-\frac{1}{2}$	0	+1
$\bar{P}_{i'}$	+1		$-\frac{1}{2}$	0	-1
Φ	0		+1	0	0
$\Lambda^{3N_c-N_f}$	$2N_c$		$-N_f$	0	0
$\Lambda'^{3N'_c-N'_f}$	$2N'_c$		$-N'_f$	0	0

(2.7)

where the normalizations of the $U(1)_A \times U(1)_V \times U(1)'_V$ charges are arbitrary. The $U(1)_R$ charges are only defined up to an addition of an arbitrary multiple of the $U(1)_A$ charges.

In the quantum theory both the $U(1)_R$ and $U(1)_A$ symmetries are anomalous. With the classical charge assignments (2.7) the $U(1)_R$ symmetry is broken explicitly by the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to the anomaly free discrete subgroup $Z_{\text{GCD}(2N_c, 2N'_c)} \subset U(1)_R$ as described above. Likewise, the $U(1)_A$ symmetry is broken explicitly by $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to anomaly free discrete subgroups $Z_{N_f} \subset U(1)_A$ and $Z_{N'_f} \subset U(1)_A$, respectively. The largest simultaneous subgroup of both Z_{N_f} and $Z_{N'_f}$ which is left invariant by the superpotential (2.6) is $Z_{\text{GCD}(N_f, N'_f)} \subset U(1)_A$. The form of the potential (2.6) may be enforced by gauging the non-anomalous discrete $Z_{\text{GCD}(N_f, N'_f)}$ symmetry if it is non-trivial, along with weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. This forbids the presence of a polynomial dependence $w(\Phi)$.

The marginal tree-level superpotential (2.6) is, up to irrelevant terms, of rather generic form within many UV completions of theories with moduli dependent masses. It requires only that the masses of the flavors of both gauge groups are moduli dependent functions, and that all flavors become massless at a single point in moduli space, here defined to be $\Phi = 0$. Importantly for the discussion of metastable dynamical supersymmetry breaking below, the superpotential (2.6) contains only marginal terms, so that any relevant mass scales must arise from dimensional transmutation. Generalizations to other gauge groups and matter contents in vector-like representations with the superpotential (2.6) are straightforward.

The classical moduli space for the theory (2.1) with superpotential (2.6) depends on the gauge group ranks and number of flavors. For $\lambda = \lambda' = 0$ the moduli space is parameterized by Φ , meson invariants $M_{ij} = Q_i \bar{Q}_j$ and $M'_{i'j'} = P_{i'} \bar{P}_{j'}$ and for $N_f \geq N_c$ and/or $N'_f \geq N'_c$ baryon and anti-baryon invariants $B_{i_1 i_2 \dots i_{N_c}} = Q_{[i_1} Q_{i_2} \dots Q_{i_{N_c}]}$, $\bar{B}_{i_1 i_2 \dots i_{N_c}} = \bar{Q}_{[i_1} \bar{Q}_{i_2} \dots \bar{Q}_{i_{N_c}]}$, and/or $B'_{i_1 i_2 \dots i_{N'_c}} = P_{[i_1} P_{i_2} \dots P_{i_{N'_c}]}$, $\bar{B}'_{i_1 i_2 \dots i_{N'_c}} = \bar{P}_{[i_1} \bar{P}_{i_2} \dots \bar{P}_{i_{N'_c}]}$ respectively. For $\lambda, \lambda' \neq 0$ the superpotential (2.6) lifts all the moduli parameterized by the mesons. The remaining moduli space has a branch parameterized by Φ . For $\Phi \neq 0$ the flavors are massive and the baryon and anti-baryon directions are lifted along this branch. For $N_f \geq N_c$ and/or $N'_f \geq N'_c$ there is a second branch of the moduli space parameterized by the baryons and anti-baryons with $\Phi = 0$. The two branches touch at the point where all the moduli vanish.

2.2 Supersymmetric vacua

The classical moduli space of vacua is lifted by nonperturbative effects in the quantum theory. Since the metastable supersymmetry breaking vacua discussed below arise for $\Phi \neq 0$, only this branch of the moduli space will be considered in detail. On this branch, holomorphy, symmetries, and limits fix the exact superpotential written in terms of invariants, to be

$$\begin{aligned}
 W = & \lambda \Phi \text{Tr} M + (N_c - N_f) \left[\frac{\Lambda^{3N_c - N_f}}{\det M} \right]^{1/(N_c - N_f)} + \lambda' \Phi \text{Tr} M' \\
 & + (N'_c - N'_f) \left[\frac{\Lambda'^{3N'_c - N'_f}}{\det M'} \right]^{1/(N'_c - N'_f)}
 \end{aligned} \tag{2.8}$$

For gauge sectors in the free magnetic range, the nonperturbative contribution refers to the Seiberg dual. Since the meson invariants are lifted on this branch, they may be eliminated by equations of motion, $\partial W/\partial M_{ij} = 0$ and $\partial W/\partial M'_{i'j'} = 0$, to give the exact superpotential in terms of the classical modulus Φ

$$W = N_c [(\lambda\Phi)^{N_f} \Lambda^{3N_c - N_f}]^{1/N_c} + N'_c [(\lambda'\Phi)^{N'_f} \Lambda'^{3N'_c - N'_f}]^{1/N'_c}. \quad (2.9)$$

The supersymmetric minima are given by stationary points of the superpotential, $\partial W/\partial\Phi = 0$, for which

$$N_f [(\lambda\Phi)^{N_f} \Lambda^{3N_c - N_f}]^{1/N_c} + N'_c [(\lambda'\Phi)^{N'_f} \Lambda'^{3N'_c - N'_f}]^{1/N'_c} = 0. \quad (2.10)$$

Physically distinct supersymmetric vacua are distinguished by the expectation value of the superpotential.

2.3 Parameter ranges for the gauge sectors

Under mild assumptions we thus end up considering two SQCD sectors, characterized by (N_c, N_f, Λ) and (N'_c, N'_f, Λ') , respectively, and superpotential couplings (2.6). Different choices may be considered here; to restrict them, it is important to note that calculable quantum corrections can be generated in two different limits.

For $\lambda_i\Phi \gg \Lambda_i$, with $\Lambda_i = \Lambda$ or Λ' , the corresponding gauge group is weakly coupled and hence generates small calculable corrections to the Kähler potential. Integrating out the massive quarks, for energies below Φ , leads to gaugino condensation, which gives nonperturbative contributions as in (2.9).

On the other hand, for $\lambda_i\Phi \ll \Lambda_i$, the corresponding gauge sector becomes strongly coupled. The calculable case corresponds to having the gauge theory in the free magnetic range. For concreteness, we choose this sector to be $SU(N_c)$ (the unprimed sector), so that $N_c + 1 \leq N_f < \frac{3}{2}N_c$.

For the (N'_c, N'_f, Λ') (primed) sector, the interesting case arises for $N'_f < N'_c$ and $\lambda'\Phi \gg \Lambda'$. Although the classical superpotential pushes Φ to zero, the primed dynamics generates a nonperturbative term which makes the potential energy diverge as $\Phi \rightarrow 0$, in agreement with the fact that $\Phi = 0$ corresponds to an enhanced symmetry point where P and \bar{P} become massless. Balancing the primed and unprimed contributions leads to a runaway direction in moduli space which will be lifted by one loop corrections. This stabilizes Φ at a nonzero value. Calculability demands working in the energy range $E \gg \Lambda'$ and $E \ll \Lambda$ so the dynamically generated scales must satisfy $\Lambda' \ll \Lambda$.

The semiclassical limit corresponds to energies $E \gg \Lambda, \Lambda'$, where both sectors are weakly coupled. Since $\Lambda' \ll \Lambda$, $SU(N_c)$ confines first when flowing to the IR. For $\Lambda' \ll E \ll \Lambda$, the primed sector is weakly interacting while the unprimed sector has a dual weakly coupled description [14] in terms of the magnetic gauge group $SU(\tilde{N}_c)$ with $\tilde{N}_c = N_f - N_c$, N_f^2 singlets M_{ij} , and N_f magnetic quarks (q_i, \tilde{q}_j) . In terms of this description, the full nonperturbative superpotential reads

$$W = m\Phi \operatorname{tr} M + h \operatorname{tr} qM\tilde{q} + \lambda'\Phi \operatorname{tr} P\bar{P} + (N'_c - N'_f) \left(\frac{\Lambda'^{3N'_c - N'_f}}{\det P\bar{P}} \right)^{1/(N'_c - N'_f)} \quad (2.11)$$

$$+(N_f - N_c) \left(\frac{\det M}{\tilde{\Lambda}^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)}.$$

Hereafter, $M_{ij} = Q_i \bar{Q}_j / \Lambda$, and $m := \lambda \Lambda$. The magnetic sector has a Landau pole at $\tilde{\Lambda} = \Lambda$.

In this description, the meson M and the primed quarks (P, \bar{P}) become massless at $\Phi = 0$. $M = 0$ is also an enhanced symmetry point since here the magnetic quarks (q, \tilde{q}) become massless.

3. Metastability near enhanced symmetry points

In this section, metastable vacua near the origin of moduli space will be shown to exist for the theory with superpotential (2.11). In section 3.1, we analyze the branches of the moduli space and determine where Coleman-Weinberg effects may lift the runaway. Next, in 3.2, we focus on the region containing metastable vacua. In 3.3, we argue that other quantum corrections are under control and do not affect the stability of these vacua. Finally, in section 3.4 the metastable vacua are shown to be parametrically long-lived.

3.1 Exploring the moduli space

Starting from the superpotential (2.11), the discussion is simplified by taking the limit $\tilde{\Lambda} \rightarrow \infty$, while keeping m fixed. The nonperturbative $\det M$ term is only relevant for generating supersymmetric vacua, as discussed in (2.9), and not important for the details of the metastable vacua that will arise near $M = 0$. Thus, for $M/\tilde{\Lambda} \rightarrow 0$ and $\Phi/\tilde{\Lambda} \rightarrow 0$, it is enough to consider the superpotential

$$W = m\Phi \operatorname{tr} M + h \operatorname{tr} qM\tilde{q} + \lambda'\Phi \operatorname{tr} P\bar{P} + (N'_c - N'_f) \left(\frac{\Lambda^{3N'_c - N'_f}}{\det P\bar{P}} \right)^{1/(N'_c - N'_f)}. \quad (3.1)$$

In this limit all the fields are canonically normalized and the classical potential is

$$V = V_D + V'_D + \sum_a |W_a|^2 \quad (3.2)$$

where $W_a = \partial_a W$, and a runs over all the fields. V_D and V'_D are the usual D-term contributions from $SU(\tilde{N}_c)$ and $SU(N'_c)$. Since both gauge sectors are weakly coupled, it is enough to consider the F-terms on the D-flat moduli space, parametrized by the chiral ring. This restriction has no impact on the analysis of the metastable vacua.

Let us study the regime $P\bar{P} \rightarrow \infty$. Then nonperturbative effects from $SU(N'_c)$ may be neglected, and the classical superpotential

$$W_{cl} = m\Phi \operatorname{tr} M + h \operatorname{tr} qM\tilde{q} + \lambda'\Phi \operatorname{tr} P\bar{P} \quad (3.3)$$

is recovered. Setting

$$W_{M_{ij}} = m\Phi \delta_{ij} + hq_i \tilde{q}_j = 0, \quad (3.4)$$

we obtain $\Phi = 0$ and $hq\tilde{q} = 0$. This implies $W_{\operatorname{tr} P\bar{P}} = W_q = 0$. The locus $W_\Phi = 0$ then defines a classical moduli space of supersymmetric vacua.

Keeping $P\bar{P}$ large, but including the nonperturbative effects from $SU(N'_c)$, $W_{\text{tr}P\bar{P}} = 0$ sets $P\bar{P} \rightarrow \infty$ and $W_\Phi = 0$ implies $M \rightarrow \infty$. Therefore the model does not have a stable vacuum in the limit $\tilde{\Lambda} \rightarrow \infty$. As discussed above, for $\tilde{\Lambda}$ finite and M large enough, the nonperturbative $\det M$ term introduces supersymmetric vacua as in (2.9).

All the F-terms are small in the limit $M \rightarrow \infty$, $\Phi \rightarrow 0$, which thus corresponds to $M_F^2 \gg |F|$. The one-loop corrections give logarithmic dependences on the fields (Φ, M) and these cannot stop the power-law runaway behavior.

Thus we are led to consider the region near the enhanced symmetry point $M = 0$. As we shall see below, this still has a runaway. Crucially, it turns out that one-loop corrections stop this runaway (this novel effect is characterized as a “pseudo-runaway”). The reason for this is that the Coleman-Weinberg formula [15]

$$V_{CW} = \frac{1}{64\pi^2} \text{Str} M^4 \ln M^2 \tag{3.5}$$

will have polynomial (instead of logarithmic) dependence. This will be explained next.

A global plot of the potential is provided in figure 1, where M has been expanded around zero as below in equation (3.8). In the graphic, the ‘drain’ towards the supersymmetric vacuum corresponds to the curve $W_\Phi = 0$.

3.2 Metastability along the pseudo-runaway direction

In the region $\Phi \neq 0$, (P, \bar{P}) may be integrated out by equations of motion provided that $\Lambda' \ll \lambda'\Phi$. This is a good description if we are not exactly at the origin but near it, as given by $\Phi/\tilde{\Lambda} \ll 1$. Taking, as before, $\tilde{\Lambda} \rightarrow \infty$ and m fixed, the superpotential reads

$$W = m\Phi \text{tr} M + h \text{tr} qM\tilde{q} + N'_c [\lambda'^{N'_f} \Lambda'^{3N'_c - N'_f} \Phi^{N'_f}]^{1/N'_c}. \tag{3.6}$$

This description corresponds to an O’Raifeartaigh-type model in terms of magnetic variables but with no flat directions.

Given that $\phi = \langle \Phi \rangle \neq 0$, we will expand around the point of maximal symmetry

$$q = (q_0 \quad 0), \quad \tilde{q} = (\tilde{q}_0 \quad 0), \quad M = \begin{pmatrix} 0 & 0 \\ 0 & 0 + X \cdot I_{N_c \times N_c} \end{pmatrix}. \tag{3.7}$$

Here q_0 and \tilde{q}_0 are $\tilde{N}_c \times \tilde{N}_c$ matrices satisfying

$$hq_{0i}\tilde{q}_{0j} = -m\phi\delta_{ij}, \quad i, j = \tilde{N}_c + 1, \dots, N_f, \tag{3.8}$$

and the nonzero block matrix in M has been taken to be proportional to the identity; indeed, only $\text{tr} M$ appears in the potential. This minimizes W_M and sets $W_q = W_{\tilde{q}} = 0$. The spectrum of fluctuations around (3.7) is studied in detail in section 4, where it is shown that the lightest degrees of freedom correspond to (ϕ, X) with mass given by m . The effective potential derived from (3.6) is

$$V(\phi, X) = N_c m^2 |\phi|^2 + \left| mN_c X + N'_f \lambda'^{N'_f/N'_c} \left(\frac{\Lambda'^{3N'_c - N'_f}}{\phi^{N'_c - N'_f}} \right)^{1/N'_c} \right|^2 + V_{CW}(\phi, X), \tag{3.9}$$

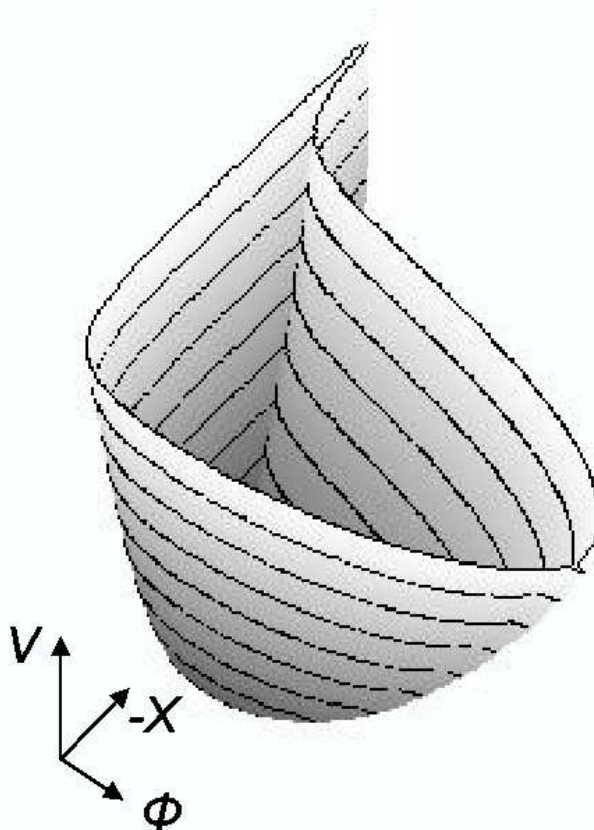


Figure 1: A plot showing the global shape of the potential. M has been expanded around zero as in equation (3.8). Note the runaway in the direction $X \rightarrow -\infty$ and $\phi \rightarrow 0$. The singularity at $\phi = 0$ and the “drain” $W_\phi = 0$ are clearly visible. Also visible is the Coleman-Weinberg channel near $X = 0$ and ϕ large, discussed later. This plot was generated with the help of [16].

where the second term comes from W_ϕ . The Coleman-Weinberg contribution will be discussed shortly.

As a starting point, set $X = 0$ and $V_{CW} \rightarrow 0$. Minimizing $V(\phi, X = 0)$ gives

$$|\phi_0|^{(2N'_c - N'_f)/N'_c} = \sqrt{\frac{N'_c - N'_f}{N_c N'_c}} N'_f \frac{\lambda'^{N'_f/N'_c}}{m} \Lambda'^{(3N'_c - N'_f)/N'_c}, \quad (3.10)$$

and since $W_{\phi\phi} \sim m$, $V(\phi_0 + \delta\phi, X = 0)$ corresponds to a parabola of curvature m . The nonperturbative term only affects ϕ_0 but not the curvature m ; this will be important in the discussion of subsection 3.4.

Next, allowing X to fluctuate (but still keeping $V_{CW} \rightarrow 0$), $V(\phi_0, X)$ gives a parabola centered at

$$X_{W_\phi=0} = -\sqrt{\frac{N'_c}{N_c(N'_c - N'_f)}} |\phi_0| \quad (3.11)$$

and curvature m . In other words, $X = 0$ is on the side of a hill of curvature m and height $V(\phi_0, 0) \sim m^2 |\phi_0|^2$.

To create a minimum near $X = 0$, V_{CW} should contain a term $m_{CW}^2 |X|^2$, with $m_{CW} \gg m$; this would overwhelm the classical curvature. As explained in section 4, the massive degrees of freedom giving the dominant contribution to V_{CW} come from integrating out the massive fluctuations along q_0 and \tilde{q}_0 . The result is

$$V_{CW} = N_c b h^3 m |\phi| |X|^2 + \dots \quad (3.12)$$

with $b = (\log 4 - 1)/8\pi^2 \tilde{N}_c [1]$, and ‘...’ represent contributions that are unimportant for the present discussion. In this computation, X and ϕ are taken as background fields. It is crucial to notice that the quadratic X dependence appears because $X = 0$ is an enhanced symmetry point.

In order to be able to produce a local minimum, the marginal parameters (λ, λ') will have to be tuned to satisfy

$$\epsilon \equiv \frac{m^2}{m_{CW}^2} = \frac{m}{b h^3 |\phi|} \ll 1. \quad (3.13)$$

In this approximation, the value of ϕ at the minimum is still given by (3.10); also, X is stabilized at the nonzero value

$$X_0 = -e^{-i \frac{N'_c - N'_f}{N'_c} \alpha_\phi} \frac{N'_f}{b h^3} \lambda'^{N'_f/N'_c} \left(\frac{\Lambda'^{3N'_c - N'_f}}{|\phi_0|^{2N'_c - N'_f}} \right)^{1/N'_c}. \quad (3.14)$$

The phases of ϕ and X are thus related by

$$\alpha_X + \frac{N'_c - N'_f}{N'_c} \alpha_\phi = \pi. \quad (3.15)$$

Inserting (3.10) into (3.14) gives

$$|X_0| = \sqrt{\frac{N_c N'_c}{N'_c - N'_f}} \frac{m}{b h^3}. \quad (3.16)$$

At the minimum, (3.13) gives

$$(m/\Lambda')^{3N'_c - N'_f} \ll (b h^3)^{(2N'_c - N'_f)/N'_c} \lambda'^{N'_f} \quad (3.17)$$

so the Yukawa coupling λ in $m = \lambda \Lambda$ must be taken small for the analysis to be self-consistent. The calculability condition $\Lambda' \ll \lambda' \Phi$ follows as a consequence of this. At the minimum, $X_0 \ll \phi_0$. The F-terms are given by

$$W_\phi \approx \sqrt{\frac{N_c N'_c}{N'_c - N'_f}} m \phi_0 \sim W_X. \quad (3.18)$$

and from (3.10) the scale of supersymmetry breaking is thus controlled by the dynamical scales of both gauge sectors. In the next subsection, the vacuum will be shown to be long-lived if (3.13) is satisfied.

Thus the model has a metastable vacuum near the origin, created by a combination of quantum corrections and nonperturbative gauge effects. The pseudo-runaway towards $X = X_{W_\phi=0}$ has been lifted by the Coleman-Weinberg contribution, as anticipated. This is the origin of the $1/b$ dependence in (3.16). The local minimum is depicted in figure 2.

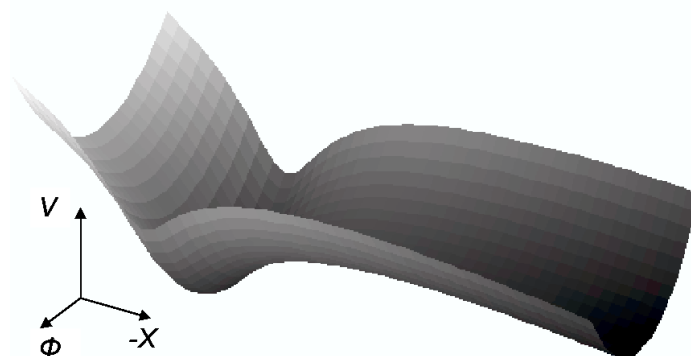


Figure 2: A plot showing the shape of the potential, including the one-loop Coleman-Weinberg corrections, near the metastable minimum. In the ϕ -direction the potential is a parabola, whereas in the X -direction it is a side of a hill with a minimum created due to quantum corrections. This plot was generated with the help of [16].

3.3 Stability under other quantum corrections

The metastable vacuum appears from a competing effect between a runaway behavior in the primed sector and one loop corrections for the meson field X . One is naturally led to ask if, under these circumstances, other quantum effects are under control. These include higher loop terms from the massive particles producing V_{CW} as well as perturbative g' corrections.

Let us first study higher loop contributions from the massive fields in (q, \tilde{q}) . They can correct the potential by additive terms of the form X^n , $n > 2$; these are automatically subleading, because $|X_0|^2 \ll m|\phi_0|$. They can also produce higher ϕ powers. However, such quantum corrections can only depend on the combination $m\phi$, and thus will be suppressed by powers of the UV cutoff Λ_0 . For instance, a quartic term would appear as $(m\phi)^4/\Lambda_0^4$. We conclude that all these effects are subleading to (3.12).

Furthermore, since nonperturbative effects from $SU(N'_c)$ were used, we should make sure that perturbative g' effects are not important. First note that the nonperturbative term in (3.9) is of the same order as the classical height of the potential $m^2|\phi|^2$ (see eq. (3.18)). It thus suffices to show that g' perturbative corrections to this height are subleading. A simple argument for this is as follows. Loops generate typical quartic terms in the Kähler potential

$$\delta K = \frac{\alpha}{\Lambda_0^2} (\Phi^* \Phi)^2 \tag{3.19}$$

which change the scalar potential by

$$\left[\frac{\alpha}{\Lambda_0^2} |\phi|^2 \right] (m^2 |\phi|^2). \tag{3.20}$$

The prefactor is parametrically small, making these contributions negligible.

3.4 Tunneling out of the metastable vacuum

This section will show that the metastable non-supersymmetric vacuum can be made parametrically long-lived by taking the parameter $\epsilon \equiv \frac{m}{bh^3|\phi_0|}$ sufficiently small. The lifetime of the metastable vacuum may be estimated using semiclassical techniques and is proportional to the exponential of the bounce action, e^B [17].

First, the direction of tunneling in field space needs to be determined. Recall that the metastable vacuum in the $(|\phi|, X)$ space lies at

$$|\phi_0|^{(2N'_c - N'_f)/N'_c} = \sqrt{\frac{N'_c - N'_f}{N_c N'_c}} N'_f \frac{\lambda'^{N'_f/N'_c}}{m} \Lambda'^{(3N'_c - N'_f)/N'_c}, \quad X_0 = -\sqrt{\frac{N_c N'_c}{N'_c - N'_f}} \frac{m}{bh^3}. \quad (3.21)$$

(The phase of ϕ , not of qualitative importance for the present discussion, has been chosen to be zero. This fixes X to be real - see equation (3.15).) For fixed X the potential has a minimum at $|\phi| = |\phi_0|$; while quantum corrections may change this value by an order one number, corrections to the curvature of the potential in the $|\phi|$ direction are negligible. This curvature is positive, and thus the potential increases as $|\phi|$ moves away from $|\phi_0|$. The field therefore does not tunnel in the $|\phi|$ direction (see figure 2). Along the X direction, however, the potential without quantum corrections near the enhanced symmetry point is like the side of a hill. For fixed $|\phi| = |\phi_0|$, the potential decreases in the negative X direction, and the classical curvature at $X = 0$ is m .

Quantum corrections are qualitatively important when $|X|$ is sufficiently small. For $|X|^2 \ll |W_X|$, their size grows quadratically as a function of X and they are sufficient to change the slope of the classical potential enough to introduce a minimum. For $|X|^2 \simeq |W_X|$, the growth of the quantum corrections is only logarithmic, and the slope of the classical potential again starts to dominate. Hence, the total potential has a peak that parametrically may be estimated to lie near

$$X_{\text{peak}} \simeq -\sqrt{|W_X|} = -\sqrt{N_c m |\phi_0|}. \quad (3.22)$$

For $|X| > |X_{\text{peak}}|$, the potential decreases as X becomes more negative until X reaches the ‘drain’ $W_\phi = 0$,

$$X_{W_\phi=0} = -\sqrt{\frac{N'_c}{N_c(N'_c - N'_f)}} |\phi_0|. \quad (3.23)$$

The direction in field space to tunnel out of the false vacuum is towards negative X with fixed $|\phi| = |\phi_0|$. It thus suffices to consider the tunneling in the one-dimensional potential, $V(X) \equiv V(|\phi_0|, X)$. Note that parametrically $|X_0| \ll |X_{\text{peak}}| \ll |X_{W_\phi=0}|$ as $\epsilon \rightarrow 0$.

For negative X , using equations (3.9) and (3.21), the one-dimensional potential may be written as

$$V(X) = \left(\frac{2N'_c - N'_f}{N'_c - N'_f} \right) N_c m^2 |\phi_0|^2 + N_c^2 bh^3 m^2 |\phi_0|^2 f\left(\frac{-|X|}{bh^3|\phi_0|}\right). \quad (3.24)$$

In the region $|X| \ll |X_{\text{peak}}|$, the function $f(x)$ is dominated by quantum corrections and may be approximated by

$$f(x) \simeq \frac{bh^3}{N_c \epsilon} x^2, \tag{3.25}$$

where a constant piece coming from the quantum corrections, again not important for the calculation of the bounce action, has been neglected. On the other hand, in the region $|X_{\text{peak}}| \ll |X| \ll |X_{W_\phi=0}|$, the constant slope of the classical potential dominates. The potential in this region may be approximated by the classical potential plus a constant contribution from the quantum corrections whose size is roughly given by the height of the potential barrier. The height of the potential barrier is, from (3.25), of order $f(X_{\text{peak}}/bh^3|\phi_0|) = 1$, and it is thus loop-suppressed compared to the overall magnitude of the potential near the metastable minimum. The potential in this region will be parametrized by a straight line

$$f(x) \simeq 1 - 2\sqrt{\frac{N'_c}{N_c(N'_c - N'_f)}}(x - x_{\text{peak}}). \tag{3.26}$$

In order to estimate the bounce action it is not appropriate to use the thin-wall approximation [17]. Instead, the potential may be modeled as a triangular barrier [18]. Using the results of [18], the value to which the field tunnels to is

$$\tilde{X} \sim -bh^3|\phi_0|. \tag{3.27}$$

Note that parametrically $|X_0| \ll |X_{\text{peak}}| \ll |\tilde{X}|$ as $\epsilon \rightarrow 0$, and that $|\tilde{X}|$ is loop-suppressed compared to $|X_{W_\phi=0}|$. The bounce action scales as

$$B \sim \frac{\tilde{X}^4}{V(X_{\text{peak}}) - V(X_0)} \sim bh^3 \frac{1}{\epsilon^2}. \tag{3.28}$$

Therefore $B \rightarrow \infty$ as $\epsilon \rightarrow 0$, and the metastable vacuum is parametrically long-lived.

The total potential $V(X)$, including the full one-loop Coleman-Weinberg potential computed numerically with the help of [16], is shown in figure 3. The program of [16] also allowed us to check numerically the previous tunneling properties.

4. Particle spectrum and R-symmetry

In this section, we discuss in more detail the particle spectrum of the model and comment on the R-symmetry properties.

The fluctuations of the fields around the metastable minimum may be parametrized following ISS,

$$\phi = \phi_0 + \delta\phi, \quad M = \begin{pmatrix} Y_{\tilde{N}_c \times \tilde{N}_c} & Z_{\tilde{N}_c \times (N_f - \tilde{N}_c)}^T \\ \tilde{Z}_{(N_f - \tilde{N}_c) \times \tilde{N}_c} & X_0 + X_{(N_f - \tilde{N}_c) \times (N_f - \tilde{N}_c)} \end{pmatrix} \tag{4.1}$$

$$q = \begin{pmatrix} q_0 + \chi_{\tilde{N}_c \times \tilde{N}_c} \\ \rho_{(N_f - \tilde{N}_c) \times \tilde{N}_c} \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{q}_0 + \tilde{\chi}_{\tilde{N}_c \times \tilde{N}_c} \\ \tilde{\rho}_{(N_f - \tilde{N}_c) \times \tilde{N}_c} \end{pmatrix}, \tag{4.2}$$

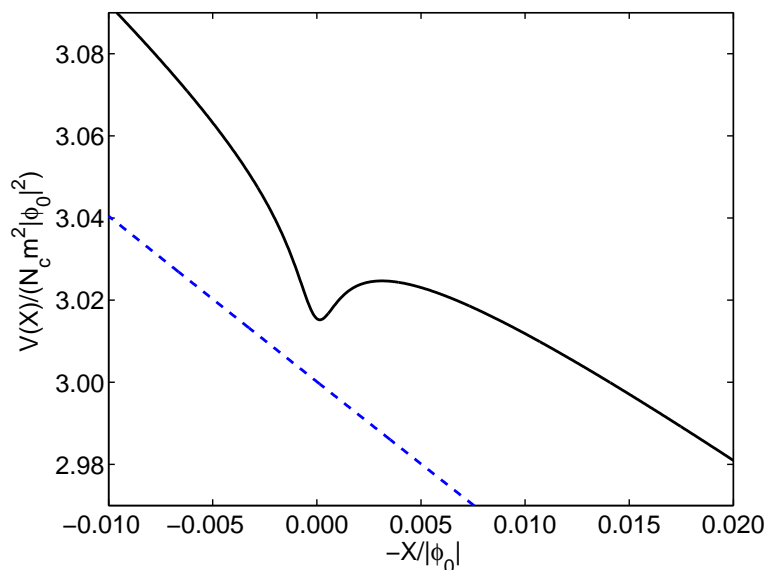


Figure 3: A plot of the classical potential (dashed line) and the total potential including one-loop corrections (solid line) for fixed $|\phi| = |\phi_0|$, where $|\phi_0|$ is the position of the metastable minimum in the ϕ -direction, defined in (3.21). In the figure, $N_f = 3$, $N_c = 2$, $N'_f = 1$ and $N'_c = 2$. The values were scaled so that the position of the “drain”, $W_\phi = 0$, equals 1 on both axes. In these units, the position of the metastable minimum is on the order of 10^{-4} . This plot was generated with the help of [16].

where $q_0 \tilde{q}_0 := -m\phi_0/h$. All fields are complex; ϕ_0 and X_0 are the values at the metastable minimum.

The relevant mass scales are

$$M^2 = 0, m^2, m_{CW}^2 = bh^3 m|\phi_0|, hm|\phi_0|. \tag{4.3}$$

The particles may be divided into three ‘sectors’ with small mixing amongst themselves. Up to quadratic order, the superpotential is

$$\begin{aligned} W = & W_{\phi\phi} \delta\phi \delta\phi + mN_c \delta\phi (X_0 + X) + m\delta\phi \sum_{\alpha=1}^{\tilde{N}_c} Y_{\alpha\alpha} + \\ & + mN_c \phi_0 (X_0 + X) + h \sum_{f=1}^{N_c} [q_0 (\tilde{\rho}Z^T)_{ff} + \tilde{q}_0 (\rho\tilde{Z}^T)_{ff} + X_0 (\rho\tilde{\rho}^T)_{ff}] \\ & + h \sum_{\alpha=1}^{\tilde{N}_c} [q_0 (\tilde{\chi}Y)_{\alpha\alpha} + \tilde{q}_0 (\chi Y)_{\alpha\alpha}]. \end{aligned} \tag{4.4}$$

The first line is related to the new dynamical field $\delta\phi$; unlike ISS, now X is not a pseudo-flat direction. The second and third lines are as in ISS.

Consider the case $N_f = N_c + 1$; the spectrum of classical masses is shown in table 1, and the spectrum of the masses including one-loop CW corrections is shown in table 2. The fields are grouped in sectors of $\text{STr}M^2 = 0$.

	Fermions			Bosons		
	Weyl mult.	mass ²	$U(N_f - 1)$	Real mult.	mass ²	$U(N_f - 1)$
$\phi, \text{tr} X$	2	$\mathcal{O}(m^2)$	1_0	1 3	0 $\mathcal{O}(m^2)$	1_0 1_0
$X_{ij} - \text{tr} X$	$(N_f - 1)^2 - 1$	0	Adj ₀	$2((N_f - 1)^2 - 1)$	0	Adj ₀
$Y, \chi, \tilde{\chi}$	1	0	1_0	1 1 4	0_{GB} 0_{NCGB} $\mathcal{O}(hm \phi_0)$	1_0 1_0 1_0
$Z, \tilde{Z}, \rho, \tilde{\rho}$	$2(N_f - 1)$ $2(N_f - 1)$	$\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	$\square_1 + \overline{\square}_{-1}$ $\square_1 + \overline{\square}_{-1}$	$2(N_f - 1)$ $2(N_f - 1)$ $2(N_f - 1)$ $2(N_f - 1)$	0_{GB} $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	\square_1 $\overline{\square}_{-1}$ $(\square_1 + \overline{\square}_{-1})$ $\overline{\square}_{-1}$

Table 1: Table showing the classical mass spectrum, grouped in sectors of $\text{Str } M^2 = 0$ for $N_f = N_c + 1$. The $\mathcal{O}(m^2)$ fields in $(\phi, \text{tr } X)$ are not degenerate. Although supersymmetry is spontaneously broken, there is no goldstino at the classical level.

	Fermions			Bosons		
	Weyl mult.	mass ²	$U(N_f - 1)$	Real mult.	mass ²	$U(N_f - 1)$
$\phi, \text{tr} X$	1 1	0 $\mathcal{O}(m^2)$	1_0 1_0	1 1 2	0 $\mathcal{O}(m^2)$ $\mathcal{O}(m_{\text{CW}}^2)$	1_0 1_0 1_0
$X_{ij} - \text{tr} X$	$(N_f - 1)^2 - 1$	0	Adj ₀	$2((N_f - 1)^2 - 1)$	$\mathcal{O}(m_{\text{CW}}^2)$	Adj ₀
$Y, \chi, \tilde{\chi}$	1	0	1_0	1 1 4	0_{GB} $\mathcal{O}(m_{\text{CW}}^2)$ $\mathcal{O}(hm \phi_0)$	1_0 1_0 1_0
$Z, \tilde{Z}, \rho, \tilde{\rho}$	$2(N_f - 1)$ $2(N_f - 1)$	$\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	$\square_1 + \overline{\square}_{-1}$ $\square_1 + \overline{\square}_{-1}$	$2(N_f - 1)$ $2(N_f - 1)$ $2(N_f - 1)$ $2(N_f - 1)$	0_{GB} $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	\square_1 $\overline{\square}_{-1}$ $(\square_1 + \overline{\square}_{-1})$ $\overline{\square}_{-1}$

Table 2: Table showing the mass spectrum, including one-loop corrections, grouped in sectors of $\text{Str } M^2 = 0$ for $N_f = N_c + 1$. Notice the appearance of the goldstino in the $(\phi, \text{tr } X)$ sector. The $\mathcal{O}(m^2)$ fields in $(\phi, \text{tr } X)$ are not degenerate; here $m_{\text{CW}}^2 = bh^3 m|\phi_0|$.

The fields $(Y, \chi, \tilde{\chi})$ form three chiral superfields, with supersymmetric masses, and hence do not contribute when integrated out at one loop. The Coleman-Weinberg potential is generated by the fields $(Z, \tilde{Z}, \rho, \tilde{\rho})$, which are the heaviest in the spectrum. Including such quantum corrections, $\text{tr } X$ acquires a mass m_{CW}^2 , while the mass of ϕ is not modified. Interestingly, at the classical level there is no massless goldstino, since the expansion is not around a critical point of the classical potential. Including quantum corrections, one of the massive fermions in the $(\phi, \text{tr } X)$ -sector becomes massless, as may be seen in table 2. A similar situation, in the opposite limit of small supersymmetry breaking, has been discussed recently in [19].

The case $\tilde{N}_c = N_f - N_c > 1$ can be similarly analyzed, and is shown in table 3.

	Fermions				Bosons			
	Weyl mult.	mass ²	$U(N_f - \tilde{N}_c)$	$SU(\tilde{N}_c)_D$	Real mult.	mass ²	$U(N_f - \tilde{N}_c)$	$SU(\tilde{N}_c)_D$
$\phi, \text{tr} X$	2	$\mathcal{O}(m^2)$	1_0	1	1 3	0 $\mathcal{O}(m^2)$	1_0 1_0	1 1
$X_{ij} - \text{tr} X$	$(N_f - \tilde{N}_c)^2 - 1$	0	Adj ₀	1	$2((N_f - \tilde{N}_c)^2 - 1)$	0	Adj ₀	1
$Y, \chi, \tilde{\chi}$	\tilde{N}_c^2	0	1_0	Adj	\tilde{N}_c^2 \tilde{N}_c^2 $4\tilde{N}_c^2$	0_{GB} 0_{NCGB} $\mathcal{O}(hm \phi_0)$	1_0 1_0 1_0	Adj Adj Adj
$Z, \tilde{Z}, \rho, \tilde{\rho}$	$2\tilde{N}_c(N_f - \tilde{N}_c)$ $2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	$\square_1 + \bar{\square}_{-1}$ $\square_1 + \bar{\square}_{-1}$	$\square + \bar{\square}$ $\square + \bar{\square}$	$2\tilde{N}_c(N_f - \tilde{N}_c)$ $2\tilde{N}_c(N_f - \tilde{N}_c)$ $2\tilde{N}_c(N_f - \tilde{N}_c)$ $2\tilde{N}_c(N_f - \tilde{N}_c)$	0_{GB} $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$ $\mathcal{O}(hm \phi_0)$	\square_1 $\bar{\square}_{-1}$ $(\square_1 + \bar{\square}_{-1})$ $\bar{\square}_{-1}$	$\bar{\square}$ \square $(\bar{\square} + \square)$ \square

Table 3: Table showing the classical mass spectrum, grouped in sectors of $\text{Str } M^2 = 0$, for $N_f > N_c + 1$. After gauging $SU(\tilde{N}_c)$, the traceless goldstone bosons from $(\chi, \tilde{\chi})$ are eaten, giving a mass $m_{\tilde{W}}^2 = g^2 m |\phi_0| / h$ to the gauge bosons. Further, from $V_D = 0$, the noncompact goldstones also acquire a mass $m_{\tilde{W}}^2$. Including CW corrections, $\text{tr} X$ acquires mass m_{CW}^2 and one of the fermions becomes massless.

The Standard Model gauge group can be embedded inside the global symmetry group of this model. In this way, renormalizable models of direct gauge mediated supersymmetry breaking may be constructed.

4.1 Breaking the R-symmetry

To have gaugino masses, any R-symmetry must be broken, explicitly and/or spontaneously [1, 19]. The low energy superpotential (3.6) has the following $U(1)_R$ symmetry:

$$R_\phi = 2 \frac{N'_c}{N'_f}, \quad R_X = 2 \frac{N'_f - N'_c}{N'_f}, \quad R_q = R_{\tilde{q}} = \frac{N'_c}{N'_f}. \quad (4.5)$$

Since the VEV's of these fields are nonzero in the metastable vacuum, the R-symmetry is spontaneously broken, and there is an R-axion a . In terms of the phase of the i -th field, the axion is

$$\phi_i = \frac{1}{\sqrt{2}} \frac{f_R}{R_i} e^{i R_i (a / f_R)}, \quad (4.6)$$

where the decay constant f_R is defined as

$$f_R = \left[\sum_i (\sqrt{2} R_i |\langle \phi_i \rangle|)^2 \right]^{1/2} \quad (4.7)$$

and R_i is the R-charge of ϕ_i . In [3] it was pointed out that if R-symmetry is broken spontaneously in an O' Raifeartaigh model, then the theory should contain a field with R-charge different than 0 or 2. This is also the case in the present situation, although our model does not contain the linear O' Raifeartaigh term.

For finite $\tilde{\Lambda}$, the $\det X$ contributions need to be taken into account, and the $U(1)_R$ symmetry becomes anomalous. Adding this term induces a tadpole for Y , which now acquires an expectation value of order

$$Y \sim \left[\frac{X_0}{\tilde{\Lambda}} \right]^{\frac{3N_c - 2N_f}{N_f - N_c}} X_0, \quad (4.8)$$

so that $|Y| \ll |X_0|$. Then the mass of the R-axion follows from

$$|W_X|^2 \sim \left| m\phi + cX_0^2 \left[\frac{X_0}{\widetilde{\Lambda}} \right]^2 \frac{3N_c - 2N_f}{N_f - N_c} \right|^2. \quad (4.9)$$

Deriving twice the cross-term, which is proportional to $\cos(a/f)$, yields the axion mass

$$m_a^2 \sim m^2 \left(\left[\frac{\lambda}{bh^3} \right]^2 \frac{3N_c - 2N_f}{N_f - N_c} \frac{\epsilon}{bh^3} \right) \ll m^2, \quad (4.10)$$

where λ is the Yukawa coupling appearing in $m = \lambda\Lambda$. Thus, R-symmetry is both spontaneously and explicitly broken.

5. Metastability near generic points of enhanced symmetry

In this section, the existence and genericity of metastable vacua near enhanced symmetry points is explored. Statistical analyses of the supersymmetry breaking scale up to date have not taken into account loop quantum effects [10], as these corrections are hard to evaluate on an ensemble of field theories. However, metastable vacua introduced by the Coleman-Weinberg potential, with all the relevant parameters generated dynamically, may change such results. Before considering the general case, let us analyze (2.5).

5.1 Non-coincident enhanced symmetry points

Consider two gauge sectors as in (2.5), with enhanced symmetry points at $\Phi = 0$ and $\Phi = \xi$, respectively. The free magnetic sector is taken to be massless at $\Phi = 0$; integrating over the other primed sector gives

$$W = m\Phi \operatorname{tr} M + h \operatorname{tr} qM\widetilde{q} + N'_c [\lambda^{N'_f} \Lambda^{3N'_c - N'_f} (\Phi + \xi)^{N'_f}]^{1/N'_c}. \quad (5.1)$$

Since metastable vacua were shown to exist for $\xi = 0$, here the discussion is restricted to the limit of ξ much bigger than all the energy scales in the problem. This is consistent with the fact that naturalness demands any relevant coupling to be of order the UV cutoff.

Introducing the notation

$$\alpha = N'_f/N'_c, \quad K = N'_c \lambda^{N'_f/N'_c} \Lambda^{(3N'_c - N'_f)/N'_c},$$

the equations of motion for ϕ and X give

$$N_c m^2 \phi = \alpha^2 (1 - \alpha) \frac{K^2}{\xi^{3-2\alpha}}. \quad (5.2)$$

$$|X| = \frac{N_c}{\alpha(1 - \alpha)} \frac{m^2 \xi^{2-\alpha}}{K}. \quad (5.3)$$

Without fine-tuning m or K , X tends to be driven away from the origin as ξ increases. The fine-tuning may be seen, for instance, from the requirement $m_{CW} \gg m$, which implies

$$m^3 \ll bh^3 \frac{K^2}{\xi^{3-2\alpha}}. \quad (5.4)$$

Although this resembles the calculability condition (3.17), now there are powers of the large scale ξ in the denominator. For ξ of order the UV cutoff, this represents a big fine-tuning, either on the coefficient K or on the small mass parameter m .

The conclusion is that, while metastable vacua can occur for far away enhanced symmetry points, this situation is not generic and requires fine-tuning. This is to be expected, once relevant parameters are allowed to appear in the superpotential.

5.2 General analysis

A generic structure in the landscape of effective field theories corresponds to a gauge theory with vector-like matter and mass given by a singlet, whose dynamics is related to another sector. The superpotential may be written as

$$W = f(\Phi) + \lambda \Phi \text{tr}(Q\bar{Q}). \tag{5.5}$$

Here, (Q, \bar{Q}) are N_f quarks in $SU(N_c)$ SQCD; $f(\Phi)$ may be generated, for instance, from a flux superpotential, by nonrenormalizable interactions [4], or, as in the case studied in this work, by another gauge sector. Next, it is required that the SQCD sector be in the free magnetic range; this is still a generic situation. The dual magnetic description is weakly coupled near the enhanced symmetry point $\Phi = 0$, where the superpotential reads

$$W = f(\Phi) + m\Phi \text{tr} M + h \text{tr} qM\tilde{q}. \tag{5.6}$$

The question that will be addressed here is: what restrictions need to be imposed on $f(\Phi)$, so that the one loop potential V_{CW} can create a metastable vacuum near $M = 0$? Since we are interested in the novel effect of pseudo-runaway directions we will demand $f'(\Phi) \neq 0$. The case $f'(\Phi) = 0$ is standard in such analyses, see e.g. [12].

As discussed in section 3, this is possible only if

$$m_{CW}^2 := N_c b h^3 m |\phi| \gg m^2 \tag{5.7}$$

where ϕ denotes the expectation value of Φ at the metastable vacuum. Further, one needs to impose that

$$h^2 |X|^2 \ll m |\phi| \tag{5.8}$$

in order for the Taylor expansion of V_{CW} around $X = 0$ to converge. Evaluating the potential as in (3.9),

$$V = N_c m^2 |\phi|^2 + |f'(\phi) + m N_c X|^2 + m_{CW}^2 |X|^2. \tag{5.9}$$

The rank condition, an essential ingredient in the discussion, just follows from having SQCD in the free magnetic range. This fixes the first term, which comes from W_M , and the block structure of the matrix M ; X was defined in (3.7).

Extremizing $V(\phi, X = 0)$ leads to

$$N_c m^2 \phi = -f'(\phi) f''(\phi)^*. \tag{5.10}$$

On the other hand, minimization with respect to X in the approximation $m_{CW}^2 \gg m^2$, gives the metastable vacuum

$$m_{CW}^2 X = -N_c m f'(\phi). \tag{5.11}$$

Notice that $m_{CW}^2 \gg m^2$ makes this value parametrically smaller than the position of the ‘drain’ $f'(\phi) + m N_c X = 0$. This ensures the stability of the nonsupersymmetric vacuum. Replacing (5.10) in (5.11) (with $m_{CW}^2 = N_c b h^3 |\phi|$) yields

$$|X| = \frac{N_c m^2}{b h^3} \frac{1}{|f''(\phi)|}. \tag{5.12}$$

It is possible to combine the conditions (5.7) and (5.8) with the values at the metastable vacuum (5.10), (5.12), to derive constraints on $f(\phi)$: (5.7) now reads

$$\frac{|f'(\phi) f''(\phi)|}{m^3} \gg \frac{1}{b h^3}, \tag{5.13}$$

while (5.8) gives

$$h^2 |f'(\phi)|^2 \ll m (b h^3)^2 |\phi|^3. \tag{5.14}$$

Summarizing, the necessary conditions for metastable vacua near $X = 0$ to exist are (5.13) and (5.14). As illustrated in the previous subsection, they require fine-tuning the coefficients of $f(\phi)$, except in the case of coincident enhanced symmetry points, where there are no relevant scales.

6. Conclusions

In this paper we constructed a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. The model consists of two $N = 1$ supersymmetric QCD sectors with flavors whose respective masses are controlled by the same singlet field. One of the gauge sectors is in the free magnetic range while the other is in the electric range. The metastable vacua are produced near a point of enhanced symmetry by a combination of nonperturbative gauge effects and, crucially, perturbative effects coming from the one-loop Coleman-Weinberg potential.

The model has the following desirable features: an explicitly and spontaneously broken R -symmetry, a singlet, a large global symmetry, naturalness and renormalizability.

There are two points that have to be stressed. First, a salient feature of the model is the existence of pseudo-runaway directions. They correspond to a runaway behavior that is lifted by one loop quantum corrections. This has not been observed before, the closest analog corresponding for example to the pseudo-moduli of [1]. It is quite plausible that this phenomenon appears in other models as well. The criterion is that the height of the potential has to be parametrically larger than the curvature, as quantified in section 3. The strength of the quadratic Coleman-Weinberg corrections is set by this height, thus introducing a local minimum of high curvature in the (otherwise) runaway potential.

In dynamical supersymmetry breaking models [20, 21], nonsupersymmetric vacua generally arise due to competing effects between a nonperturbative runaway and a classical term in the superpotential, as in the (3,2) model [22]. Our analysis shows that it is possible to stabilize such runaways even without tree-level terms, provided that one is close to certain enhanced symmetry points.

The second feature worth emphasizing is the connection between enhanced symmetry points in gauge theory moduli spaces and metastable dynamical supersymmetry breaking. There are reasons to believe that such vacua are generic. At the field theory level this is associated to the fact that a nonzero Witten index [23] may still allow an approximate R-symmetry [24]. While dynamical ISS models are not hard to construct, in general these mechanisms involve discrete R-symmetries [4]. This is very suppressed in the landscape of string vacua, corresponding to a high codimension locus in the flux lattice [25]. On the other hand, the construction presented here does not suffer from the previous difficulty. Therefore, it would be interesting to study how statistical estimates of the scale of supersymmetry breaking change, once the model is embedded in string theory.

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